Environmental Impact of Blasting Planning to Comply

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Abstract

Blasting operations are arguably one of the most contentious areas of surface mining. Public relations are easily soured at the mere hint of proposed plans, increased numbers of nuisance complaints only lead to tighter restrictions and an ever growing band of lawyers are always ready to advise today's litigious society. Such opposition can easily render mineral extraction untenable.

Many operators carry out routine monitoring as required by Mineral Planning Authorities. However in so doing they only concentrate on the observed Maximum Peak Particle Velocity (PPV) being below the statutory limit and pay little or no attention to the measurable parameters that determine this value (e.g. scaled distance, burden, detonator type, blast type or rock type). Such practice is "monitoring to comply". What is needed is for the operators to adopt a methodology that clearly demonstrates that the operator is "planning to comply". The dedicated recording and analysis of blast vibration data is a critically important part of today's 'best-practice' methodology. This can only be achieved by constantly updating regression curves using compatible charge weights and vibration predictions determined from the most current data.

This paper seeks to illustrate a number of the 'pit-falls' by highlighting the opposing philosophies of 'planning for compliance' and 'monitoring for compliance' and why the simple agglomeration of data can soon lead to an unworkable situation. It examines the concept of 'Likelihood' as an approach to diagnose data-set parity and promotes Likelihood Ratio Testing as an efficient method to improve predictive capacity. The aim throughout being to promote strategies and methods that are accountable, quantifiable, confer high levels of openness and improve efficiencies whilst seeking to minimize environmental impact.

Introduction

For most UK surface mine and quarrying operations where blasting takes place, a vibration limit will be set by the local Mineral Planning Authority. Originally intended to prevent property damage, nowadays these are increasingly employed as an attempted measure to minimize the human nuisance factor. The values imposed are far lower than those based on damage criteria, but are still above human perception levels and as a consequence complaints still arise.

The statutory vibration limit will commonly appear on a sites operational license as a form of words stating that blasts must be planned so that:

" X% of all blasts are to be below Y mm/s as recorded at the nearest occupied premises."

Where:
X = an imposed upper confidence limit (typically 95 or 98%)
Y = a maximum vibration value (typically 6mm/s but maybe lower)

In order to demonstrate compliance, the monitoring and recording of blasting events are obviously critical. Well-maintained monitoring records must be held to preserve operational licenses (and also are a vital defence for today's litigious society). However this is where pit-falls can arise for the unwary and seemingly compliant operators can easily find they are facing conflict.

Scaled-Distance Regression

The expected level of ground vibration from pattern blasting is generally arrived at by empirically equating peak particle velocity with a scaled distance into a bivariate expression [1,2].
To apply the principle, the scaled-distance value for any location is first calculated by:

$$SD = D \times W^{-0.5}$$

Where:
SD = scaled distance ($m.kg^{-0.5}$)
D = separation distance, blast to receiver (m)
W = maximum instantaneous charge weight (MIC) (kg)

The empirical relationship then follows as:

$$V = A(SD)^{B}$$

Where:
V is the maximum peak particle velocity (mms^{-1})
A, B are dimensionless site factors.

The site factors *A* and *B* allow for the influence of the local geology on vibration attenuation as well as for the geometrical decay of the seismic waves. The values of *A* and *B* are derived for a specific site by least-squares regression analysis of the logarithmic plot of peak particle velocity against scaled distance. This results with a mathematical best-fit straight line ($y = mx + c$) where *A* is the peak particle intercept value at unity scaled- distance and *B* is the slope of the regression line.

Monitoring to Comply

Many operators, well aware of their obligations, dutifully carry out regular vibration monitoring in accordance with their statutory requirements and indeed will quite readily cite how important the procedure is. Here is a typical scenario: Prior to a blast, the 'nearest occupied premises' (often the same location) is visited and a seismograph positioned to capture the days event(s). Post blast, the seismograph is returned, downloaded and the resulting readings verified for compliance. Having then proven compliance (always), the results are then duly noted and archived; end of story.

This procedure can be regarded as 'Monitoring for Compliance'. Superficially, it may appear that nothing is wrong with this strategy and that maybe the case until the day a problem arises. For operators who normally experience a relatively harmonious existence, the sudden arrival of blasting complaints together with maybe the threat of litigation, for whatever reason, will appear alarming. The full implication may only become apparent following a review of the past blasting events when an updated scaled-distance regression model is constructed that reveals the true magnitude of the problem and its full ramifications.

Figure 1 illustrates an extreme example of a scaled-distance regression model for a site that has been 'monitored for compliance'. Visual inspection of the regression curve immediately highlights the problem. As practically all the data has been recorded at the same location, the range of the results (in terms of both scaled distance and PPV) is so small that the data points present themselves as a 'ball' or 'clump'. This in turn provides minimal correlation evidence for regression modeling due to the dataset's clustered form. Therefore as a consequence the mathematical 'best-fit' (least-squares regression) mean trend line actually displays a positive slope! This is completely illogical with respect to the entire scaled-distance concept, but is numerically correct for the available data.

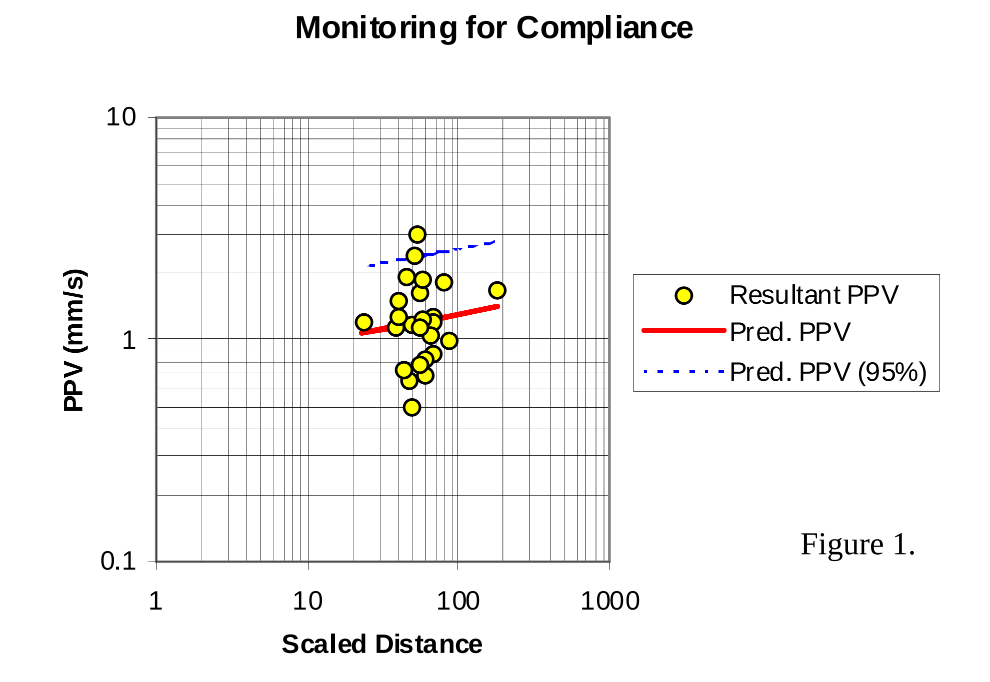

Figure 1.

The fundamental (and common) mistakes that allow such a situation to manifest are that:

- The regression curve is not being updated on a regular 'blast-by-blast' basis
- Only one seismograph is been employed at any one time
- Little variation is being introduced into the model in terms of scaled-distance and PPV
- The scaled-distance model is not being used for design purposes
- Nothing is being learnt from the blasting process

The gravity of such a situation must be stressed because such a case would be indefensible in a court of law as in no way can it be demonstrated that any of the blasting events are being specifically designed to comply with the sites operational license; compliance is in fact being achieved by chance rather than by design.

Planning to Comply

The pitfalls discussed previously can easily be avoided by adopting a properly designed and comprehensive monitoring strategy. In fact the benefits may go well beyond meeting compliance obligations as lower operating costs, increased efficiencies and improved productivity may indeed become possible through the application of more advanced analytical approaches. This procedure can be regarded as ‘Planning to Comply’.

Figure 2 illustrates an example of scaled-distance regression model for a site where blasts have been designed so as to fulfill the legal obligation by ‘planning to comply’. Here the regression curve describes a true linear relationship conforming to the scaled-distance concept. In contrast, whilst the ‘nearest occupied premises’ is still being diligently monitored for compliance, the use of further seismographs positioned at varying distances balances the logarithmic distribution and maximises the effective correlation range of the dataset. This results in a model from which license compatible charge weights and realistic vibration predictions can not only be made with genuine confidence, but they can also be accounted for and verified if ever the situation should arise.

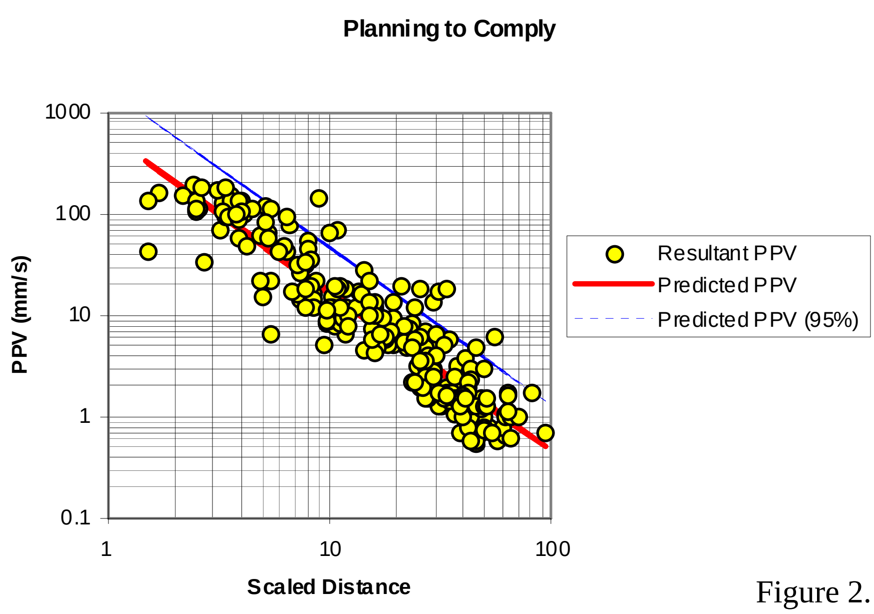

Figure 2.

Data Volume

As previously described a properly implemented monitoring strategy is an essential part of the best practice methodology for today’s blasting operators. By allowing regression curves to be constantly updated, compatible charge weight determinations and vibration predictions are derived from the most current data. However, paradoxically the point can arise when so much data becomes agglomerated that subtle variations can pass unobserved. The results of changeable blast designs, inconsistent geology [3] and variable monitoring location response [4] can soon begin to impart an adverse effect on the forecasting capability of the regression curve due to a deterioration of the standard error statistic. As this directly influences the confidence level, the effects can soon lead to a steady reduction in permissible charge weights and a decrease in vibration prediction capacity. An example of this can be seen in Figure 3, which displays the scaled-distance regression curve for a large opencast coal site. This model contains 2700 data points from 5 blasting horizons, which leads to a poor standard error (0.48) and as a direct consequence presents limited predictive capability.

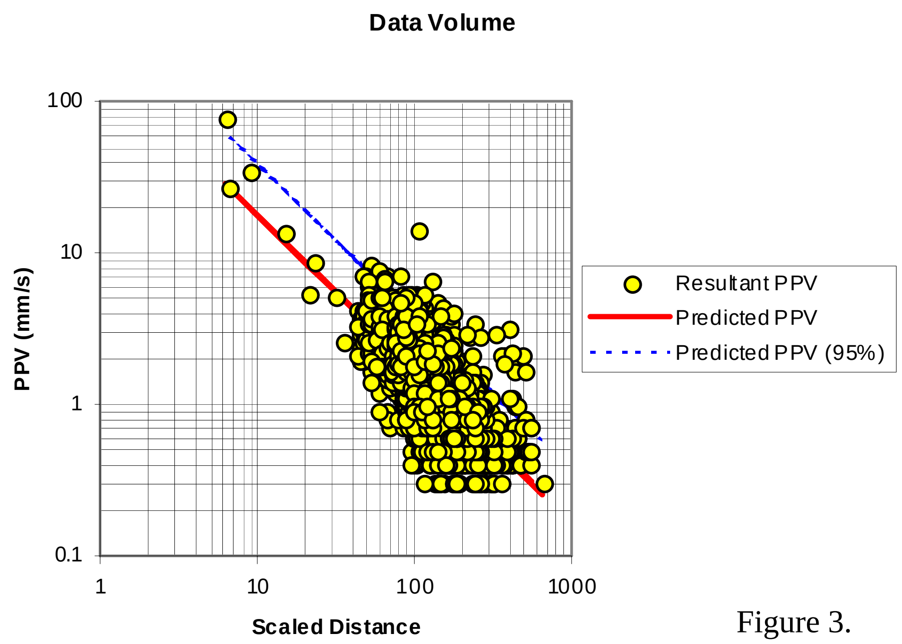

Figure 3.

The simple solution would be to only compile like-for-like blasting returns, but this prospect can quickly become unworkable due to the number of potential variables. In addition it is very difficult to spot subtle variations by simply examining a standard scaled-distance curve. This is due mainly to the intrinsic data scatter that is always present together with the logarithmic nature of the regression curve. If indeed a variation is thought to be apparent, how can the degree and magnitude of its influence to be quantified? The solution to this conundrum may not actually lie in terms of ‘probability of the forecast’, but in terms of the ‘likelihood of the outcome’.

Probability versus Likelihood

The concept of likelihood is very closely related to the concept of probability that underlies the fundamentals behind regression analysis, the previously described method for blasting parameter determination. Sir Ronald Aylmer Fisher first introduced the concept of likelihood in 1921 and refined it in 1925 when he hypothesized and prove that the likelihood of a parameter is proportional to the probability of the data and it gives a function, which usually has a single maximum value, which he called ‘Maximum Likelihood’ [5].

The language of probability is used when events are observed. For example, in a simple coin tossing experiment, the probability of observing heads is 0.5 for every toss (for an unbiased coin). Or in the blasting world, given a Maximum Instantaneous Charge (MIC) of (X) kg, for a location situated at (Y) m distance, there will be a mean probability of (Z) mm/s PPV being recorded according to scaled distance regression analysis.

When we look at the probability of observing events, be it coin tossing or vibration prediction we are in fact implicitly assuming some form of model. In our world, few things have fixed, absolute probabilities, (this is most certainly the case within the blasting engineering), however many of the aspects we observe are not truly random. This is explained by the notion of conditional probability. If we know none of the details of the blast, we would go on one assumption, but if we knew the specifics, our forecast may change dramatically. Conditional probability allows us to incorporate potentially important variables into our probability forecasts. This now leads to the concept of likelihood. If we have a definition of probability as being that ‘knowing the parameters we can make a prediction of the outcome’ then the definition of likelihood is that ‘knowing the outcome we can make an estimation of the parameters’. It then follows for it to be natural to use the parameter value(s) that are most likely given the data that has been observed, this is known as the maximum likelihood estimate(s).

Maximum Likelihood Estimation

In relation to blasting practices, we make our predictions based upon a set of solid assumptions relating to the parameters within control i.e. Maximum Instantaneous Charge weights (MIC), burden, spacing, delay intervals, decking, initiation sequence etc. This is because we are interested in the probability of a certain outcome occurring i.e. PPV levels within planning limits. However, in the analysis of blasting data, we have already observed the data and those results are now fixed. That element of probability has now ceased to exist. What we need to determine are the likelihood of the model parameters that underlie the data.

For example, consider the following simple blasting scenario, if a blast consisting of a MIC of 100kg was detonated and produced a resultant reading of 2.5mm/s on a seismograph positioned 300m away was followed by another blast in approximately the same location, again with a MIC of 100kg and a seismograph positioned 300m away, it will be more probable that a resultant reading of 2.5mm/s (or a reading close to 2.5mm/s) will again be experienced rather than a result of say 5, 0.5 or 50mm/s. With this in mind, if a resultant reading of 2.5mm/s was reported from a seismograph positioned 300m from a similar blast, it would be a logical deduction to claim that the most likely MIC was 100Kg. These are logical assumptions of both probability and likelihood.

As mentioned previously, a parameter value that is most likely given the observed data, a Maximum Likelihood Estimation (MLE), can be determined. So continuing with the previous example, a resultant reading of 2.5mm/s, positioned at 300m from a blast of 100Kg MIC can be represented by a single estimated figure and similarly so can any subsequent blast. Such MLE values will rarely be seen to give exactly the same value as some variability due to random data scatter will always be present, but if indeed the subsequent blasts were carried out in the same manner as the first then the degree of variability will be seen to be small and statistically insignificant. If on the other hand the

MLE values for two blasts were seen to be dramatically different, then this would indicate that some difference exists between the blasts. In other words a difference exists in the assumed underlying conditional probability that the two blast models are the same. This may in fact be something we are aware of e.g. one of the blasts was single decked whilst the other was multi-decked (or indeed any other design variation) or it might be the consequences of a further, as yet un-disclosed variable imparting a contributory effect e.g. geology. More than likely any number of differences between blasts will exist; identifying which are the significant contributing factors is the problem.

Parameter Estimation

The procedure of least squares estimation is broadly known and utilized. In particular it is applied to scaled-distance regression curves and used to estimate the parameters for slope and intercept as previously described. Another way to approach estimation is to specify how likely are the parameter values given the data observed and then to maximize that likelihood function. Where the least squares method is used, and the normal distribution is appropriate for scatter about the line, then the two methods are identical: maximizing the likelihood is the same as minimizing the sum of the squares of the residuals.

Least Squares Estimation:

$$y_i = ax + b + \varepsilon_i$$

where:
i = 1 to n
ε_i = random variables, assumed to be normally distributed and have an expectation of 0

Residual Sum of Squares:

$$rss = \sum_{i=1}^{n} (y_i - (ax+b))^2$$

where:
rss = residual sum of squares

Maximum Likelihood Estimation:

$$M.L. = -\frac{\sum_{i=1}^{n} (y_i - (ax+b))^2}{2\hat{\sigma}^2}$$

where:
M.L. = Maximum Likelihood
$\hat{\sigma}^2$ = biased variance

Likelihood Ratio Test

Scaled distance model fitting provides a framework from which we cannot only just estimate the maximum likelihood for parameters; we can also test whether or not they are significantly different from other values. Suppose that data is available from two blast datasets, for example from two blast sites. It is possible either to use a different pair of parameters (slope and intercept) for each dataset – a total of four parameters. It is also possible to use the same pair of parameters for both datasets – only two parameters. This is appropriate if the blast sites are similar. It is important to have a formal test to determine if 4 or 2 parameters are required. Here the likelihood ratio test will be used.

Formally the test is of:

Null hypothesis (H0): The datasets are similar in that the parameters for slope and intercept are the same for both ($a_1=a_2$ and $b_1=b_2$).

Alternative hypothesis (H1): The datasets are different in that there are separate values for the two parameters - $a_1 \neq a_2$ or $b_1 \neq b_2$ or neither equal.

The likelihood will be larger (and so the sum of squares of residuals smaller) if 4 parameters are fitted rather than just 2. If the two blast sites are not similar then the increase in the likelihood (decrease in the sum of squares of residuals) will be large. On the other hand, if the blast sites are similar then there will only be a small increase in likelihood. This is formalized by considering the ratio of the likelihoods for the 4 and 2 parameter cases (see Silvey 1970 for statistical details) [(6)].

Twice the difference in the log-likelihoods, sometimes called the deviance, turns out to be the difference in the residual sums of squares of models. Under the null hypothesis that the 2-parameter case is the correct one, this quantity (the deviance) will be distributed as a Chi Square variable with 4-2 = 2 degrees of freedom.

$$\text{Log-likelihood} = 2(LL_{(H0)} - LL_{(H1)})$$

By multiplying the difference between the likelihoods by a factor of 2, the quantity will be distributed as the familiar Chi Square statistic. This can then be assessed for statistical significance by using standard Chi Square significance levels. The degrees of freedom for the test will equal the difference in the number of parameters being estimated under the alternative and null models. For most blasting cases, 2 parameters will be estimated under the null and 4 under the alternative, therefore the Chi Square will have 2 degrees of freedom.

The following two case studies illustrate applications of the Likelihood Ratio Test to blast vibration analysis and demonstrate that real and accountable improvements that are possible.

Case Study 1

A series of test blasts were initially conducted prior to the commencement of open cast coal extraction at a site in West Yorkshire, England. Initially, blast designs and vibration predictions were based upon these findings, however the subsequent monitoring of production blasts indicates the existence of a discrepancy with the earlier findings and questions the validity of the test blast data.

The regression curves for the data can be seen in Figure 4. Initial visual inspection supports the existence of a discrepancy as the presence of two data sub-sets can be clearly seen. However, what is not clear is an indication as to what degree the data sets can be labelled different.

The results of subjecting the data to a likelihood ratio test can be seen in the lower right of Figure 4. The test derives a Log-Likelihood Ratio of 5.68, which when compared to the Chi Squared distribution indicates a percentage chance of difference of approximately 94%. In other words there is an approximate 94% chance that the test blast and production blast data are different from each other. This will come as no real surprise as the existence of a difference is easily appreciated way before the application of the Likelihood Ratio test, but what the test has provided is a credible statistic as to that degree of difference and justification for the removal of the test blast data from the sites database. The alternative would be to continue to include the test blast data and as a consequence, this would hinder the predictive performance of the model by an un-necessary degradation of the standard error statistic.

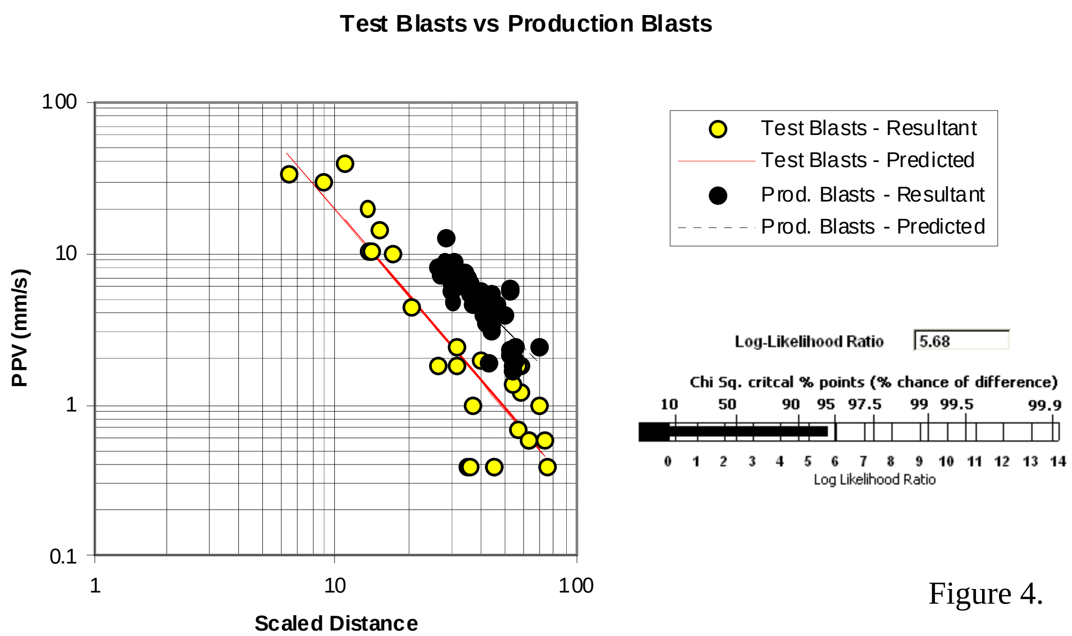

Figure 4.

Case Study 2

Blasting operations are being carried out within three defined areas at a limestone quarry in Derbyshire, England. The individual results of monitoring the areas 'Northern', 'Western' and 'Southern' can be seen in Figure 5 together with the best-fit prediction for the combined dataset.

Area variations are of great importance to the operators, particularly in the Southern area due to the close proximity of a unique cave network. This cave network being in fact one of the most heavily protected archaeological and geological sites in Britain, registered as a Geological Site of Special Interest, a Scheduled Ancient Monument, a Conservation Area, part of an Area of Local Landscape Significance and currently is being assessed for World Heritage Status. As a consequence, blast regression models for the site need to be maintained as accurately as possible.

The method adopted to analyse this data in order to diagnose any area variation, was to first subject the Northern and Western datasets to a likelihood ratio test in order to determine their level of variation. The reasoning being that if this test reports an insignificant level of variation, then these datasets are to be combined and in turn this combination dataset could then be compared with the remaining Southern data set where the potential for variation is the greatest. If on the other hand a significant difference is reported between the Northern and the Western datasets, then each dataset is to be tested against each other individually in order to explore all the possible permutations.

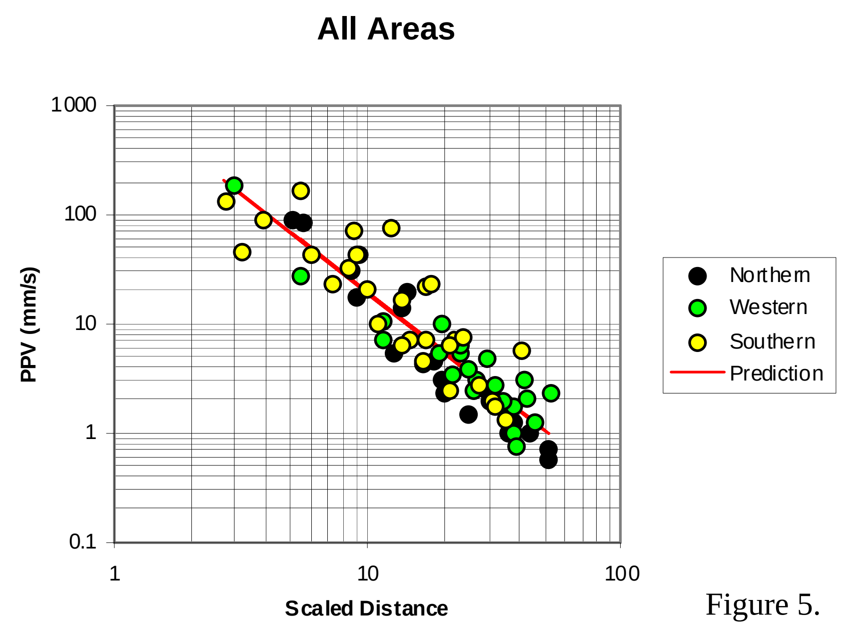

Figure 5.

The comparison between Northern and Western datasets can be seen in Figure 6 together with the results of likelihood ratio test. The results of the test do indicate the presence of a difference between these two sets by returning a value of 0.37; however the percentage chance of difference this reflects is approximately only 12% and is regarded as being statistically insignificant. On the basis of this, the Northern and Western datasets were then combined and this new dataset compared against the Southern area data.

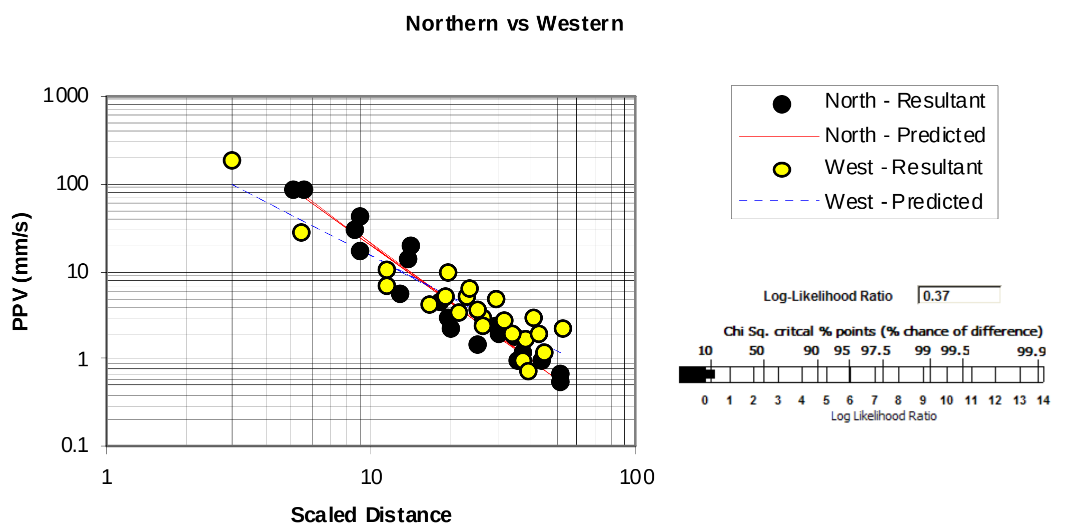

Figure 6.

The findings of the second test can be seen in Figure 7, with the results of the test this time indicating the presence of a significant difference. The value of 4.05 equates to a percentage chance of difference of approximately 80%, a significant finding. It is common in statistical assessments to regard the 90% level to be the minimum threshold beyond which the detection of a difference can be confidently reported, however due to the sensitive nature of the surrounding environment in question, it is strongly viewed that the blasting being carried out in this area and its respective data should be treat as a separate entity and not be allowed to be influenced by the blasting activities elsewhere within the site.

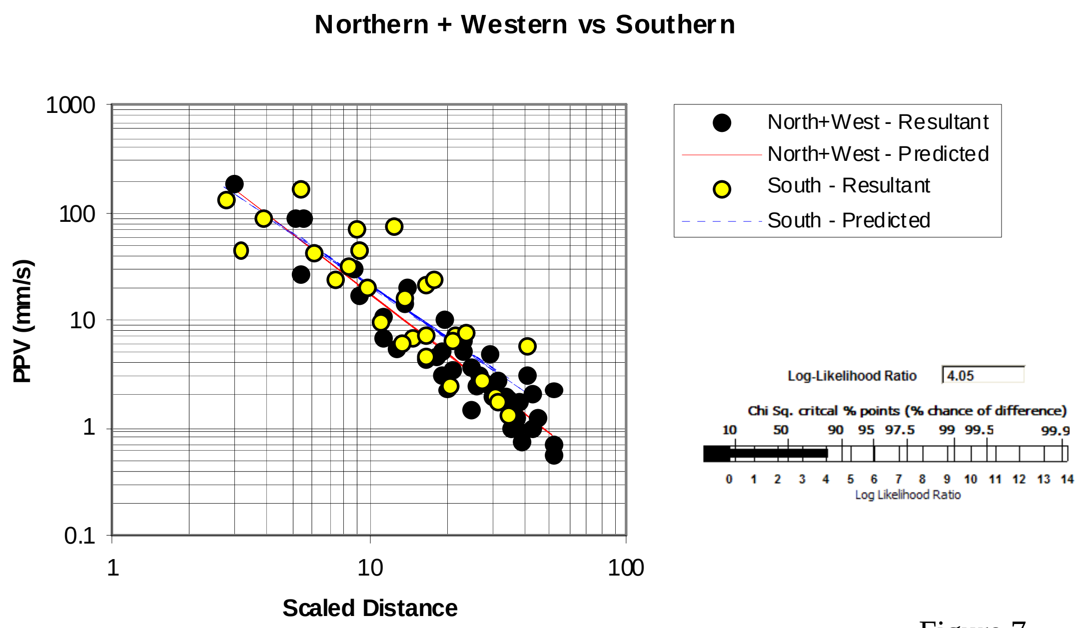

Figure 7.

Two facts are interesting to note in this case. Firstly, at no time during the exercise can any real difference between the areas be detected through visual inspection of the regression curves. Looking at Figure 5, one might say that the Southern data appears to contain a slightly higher degree of scatter, however, truthfully speaking this forms no real evidence to support any conclusion. Yet in spite of this, the likelihood ratio test has been able to deduce that a subtle variation does in fact exist.

The second interesting fact to note is that the blasts carried out in all three areas were all designed to the same specification. Therefore the difference that the test reports is entirely due to some other factor, attributable to the area that is imparting a discernable influence. In this case that factor is most likely an effect caused by the presence of the cave system and solution channels, as the true extent of the underground network is unknown. (To further investigate the location, Micro-Gravity surveys have been undertaken in the area adjacent to the caves and the results indicate a series of anomalies, which might indicate the presence of in-filled solution cavities.) Although this factor is numerically un-quantifiable, the results of the test work indicate the effect of its presence.

Case Study 3 – Location Response

All the blasting operations at a limestone quarry in North Yorkshire, England are monitored by the site operators as a mandatory requirement at the 'nearest occupied premises' as defined by the Mineral Planning Authority. The property in question is the end house of a small terraced row, known as White Row, located on the outskirts of the nearby village. The site has a planning requirement to maintain that stipulates that "95% of all blasts are to be below 8mm/s" recorded at this location. Although the planning requirement is to observe a limit of 8mm/s, the site operators run a 'good neighbour policy' by operating a self-imposed, in-house limit of just 1mm/s. A review of their past records indicates a good adherence to this policy and in those cases when 1mm/s is exceeded, the resultant PPV is still way below the actual 8mm/s requirement. However not all is as it seems, because occasionally complaints due to ground vibration are received from residents who live further away, yet an inspection of the recording made at White Row will indicate a resultant PPV in line with their working practice.

To investigate this anomaly, a second seismograph was operated in the centre of the village, positioned adjacent to the local War Memorial. The findings were analysed using a modified locational scaled-distance model [6] to diagnose the monitoring location response and by likelihood ratio test to statistically quantify any degree of variation if found present.

The results of the monitoring at the War Memorial can be seen in Figure 8 juxtaposed with the general site trend derived from the regression analysis of all the available site related data. As can be seen from the graph, the modified location trend line for the location is positioned above the general site trend line which might indicate a higher than average location response, but may in fact just be an apparent effect caused by random data scatter. The results of the likelihood ratio test can be seen in the right of Figure 8 and as can be seen, the test has been applied twice. The first test (Ratio 1) is a comparison test between the unmodified locational trend (the mathematical best fit trend line for the locations dataset) and the modified locational trend line (the pseudo trend line orientated parallel to the general site trend). This is carried out in order to assess the impact of adopting the modified trend line on the dataset's Maximum Likelihood Estimate by looking at the degree of the associated error involved. By adopting the modified trend line, the variance component contributing to the dataset's MLE is effectively altered. If the degree of error associated with this process is insignificant we can proceed using the modified trend line, if the degree of error is large then the modified locational scaled-distance model being employed will not be valid. However it is usually found that when examining datasets recorded at distant locations, the limited range of distances presented by the movement of the various blast locations fails to provide adequate correlation evidence, as the data points will tend to cluster and as a consequence the trend line can by literally placed in any position without any real negative effect. The second test (Ratio 2) is a comparison between the location dataset (using the modified trend line) and the overall general site dataset. This is carried out in order to determine whether any apparent monitoring location response is an effect of just random data scatter or whether there is an underlying discernable factor, unique to the location, which is imparting an effect.

The result of the first test (Ratio 1) for the War Memorial does indicate a small percentage chance of difference, however the level is insignificant (levels in excess of 90% chance would classified significant) and the use of the modified trend line is upheld. The result of the second test returns a Log-Likelihood Ratio of 0.84, which approximately equates to 35% chance of difference. Again this is deemed insignificant and the conclusion drawn is that the apparently slightly higher than average response of the location is primarily the consequence of random data scatter.

In comparison, the results of the monitoring at White Row can be seen in Figure 9. The result of the first test (Ratio 1) indicates that no real difference will be incurred by the assumption of the modified trend line. This is not really surprising considering the clustered nature of the data. (In reality, the mathematical best-fit line for this dataset produces a trend line with a positive slope and is one of the reasons why the modified locational scaled-distance method is being employed). Looking at the result of the second test (Ratio 2) presents a different picture. The test returns a Log-Likelihood Ratio of 5.97 that corresponds to a percentage chance of difference in excess of 98%. In other words there is at least 98% chance that the location response is unique. Evidence that leads to the conclusion that the monitoring location at White Row is in someway isolated or dampened in its response, which culminates in the regular reporting of readings far lower than average. This leaves the operators in an awkward position as they have a mandatory requirement to monitor at this location, but also that the readings they obtain are influenced by a unique location response and do not represent the seismic propagation experienced by the greater surrounding area. As a consequence the operators have adopted a modified site regression model that accounts for the low response at White Row and in turn produces a more realistic vibration predictions for the remainder of the village.

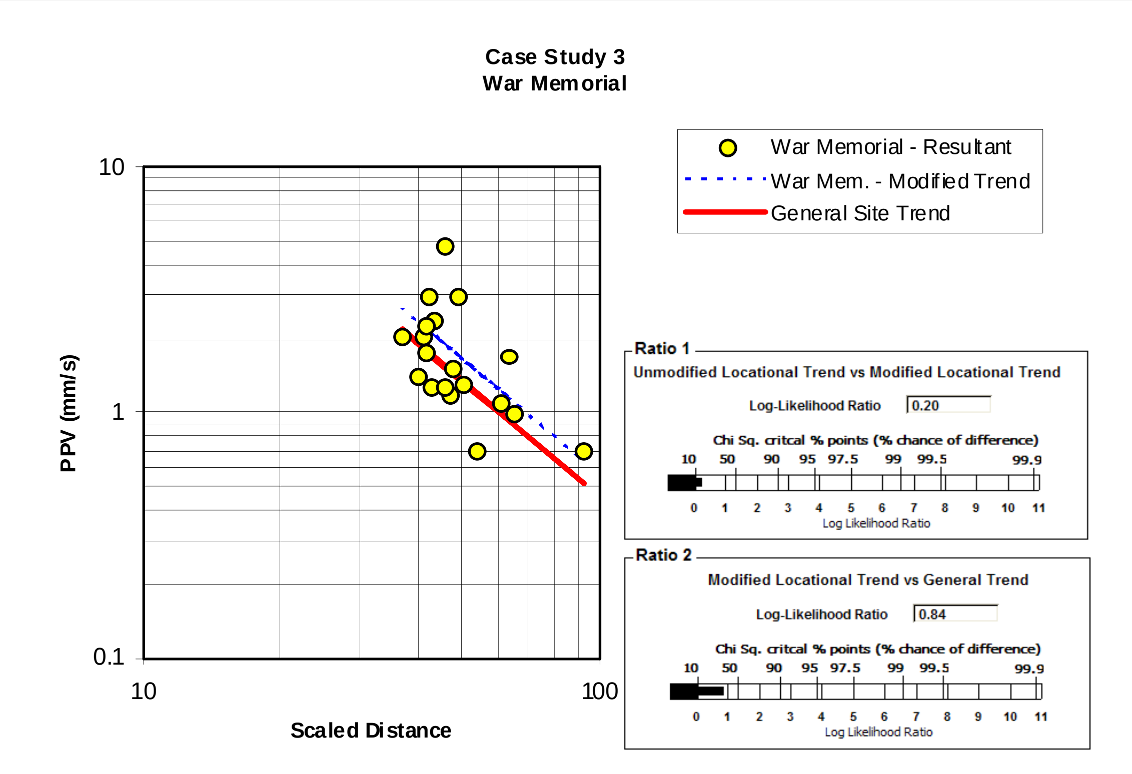

Figure 8.

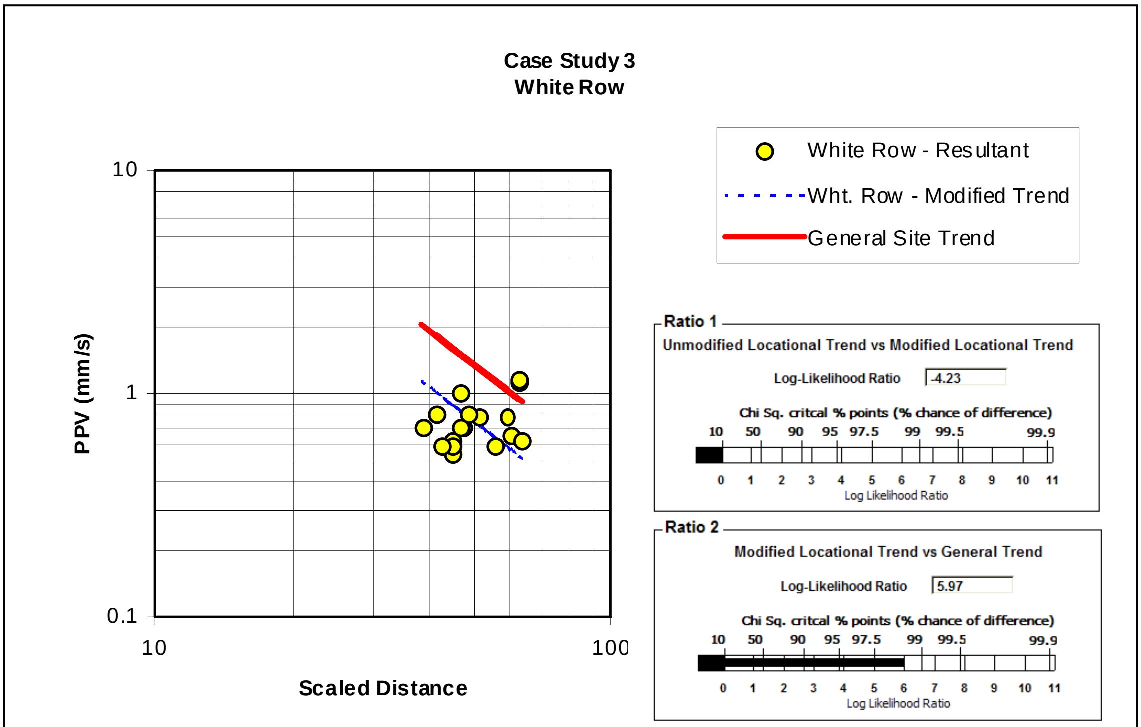

Figure 9.

Conclusion

A properly designed and implemented vibration monitoring strategy forms a critical part of the blasting process for today's mineral industry. As such it must be ensured that this is stringently carried out in order for a quarry or surface mine operator to be able to demonstrate to a mineral planning authority that they are 'planning to comply' rather than simply 'monitoring to comply'.

Whilst following such a strategy, efficient blast vibration analysis can be performed through the application of the Likelihood Ratio test (as modified for use in the blasting industry). By doing so operational and environmental parameters can be optimised by excluding data from the modelling process that has quite simply been found to be significantly different. This method is a quantifiable and auditable approach that confers high levels of openness and transparency, the end result being truly accountable blast vibration analysis.

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